Clearing Restarting Automata

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- The new model can be learned very efficiently from positive examples and its stronger version enables to learn effectively a large class of languages.
- We relate the class of languages recognized by clearing restarting automata to the Chomsky hierarchy.

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  - I is a finite set of *instructions* (x, z, y), $x \in LC_k$, $y \in RC_k$, $z \in \Sigma^+$,
    - left context $LC_k = \Sigma^k \cup ¢.\Sigma^{\leq k-1}$
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  - The special symbols: $¢$ and $\$$ are called *sentinels*.
  - The *width of the instruction* $i = (x, z, y)$ is $|i| = |xzy|$.

Definition

- A word $w = uzv$ can be *rewritten* to uv ($u\underline{z}v \vdash_M uv$) if and only if there exist an instruction $i = (x, z, y) \in I$ such that:
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- The *k-cl-RA*-automaton M *recognizes* the language $L(M) = \{w \in \Sigma^* \mid M \text{ accepts } w\}$.

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- $\mathcal{L}(cl\text{-}RA) = \bigcup_{k \geq 1} \mathcal{L}(k\text{-}cl\text{-}RA)$.
- **Note**: For every *cl-RA* M: $\lambda \vdash^{*}_{M} \lambda$ hence $\lambda \in L(M)$. If we say that *cl-RA M* *recognizes a language L*, we mean that $L(M) = L \cup \{\lambda\}$.

Motivation

- This model was inspired by the *Associative Language Descriptions* (*ALD*) model:
  - By Alessandra Cherubini, Stefano Crespi-Reghizzi, Matteo Pradella, Pierluigi San Pietro.
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- The simplicity of *cl-RA* model implies that the investigation of its properties is not so difficult and also the learning of languages is easy.
- Another important advantage of this model is that the instructions are human readable.

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- For instance:
  - $aaaabbbb \vdash^{R1} aaabbb \vdash^{R1} aabb \vdash^{R1} \underline{ab} \vdash^{R2} \lambda$.
- Now we see that the word *aaaabbbb* is accepted because $aaaabbbb \vdash^{*}_{M} \lambda$.

Some Theorems

- Error preserving property: Let $M = (\Sigma, I)$ be a *cl-RA-automaton* and u, v be two words from Σ^*. If $u \vdash^*_M v$ and $u \notin L(M)$, then $v \notin L(M)$.
  - **Proof.** $v \in L(M) \Rightarrow v \vdash^*_M \lambda \Rightarrow u \vdash^*_M v \vdash^*_M \lambda \Rightarrow u \in L(M)$. ■

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  - **Proof**. $v \in L(M) \Rightarrow v \vdash^*_M \lambda \Rightarrow u \vdash^*_M v \vdash^*_M \lambda \Rightarrow u \in L(M)$. ■
- Observation: For each finite $L \subseteq \Sigma^*$ there exist *1-cl-RA*-automaton M such that $L(M) = L \cup \{\lambda\}$.
  - **Proof**. Suppose $L = \{w_1, \ldots, w_n\}$.
    Consider $I = \{(¢, w_1, \$), \ldots, (¢, w_n, \$)\}$. ■

Some Theorems

- Theorem: $\mathcal{L}(k\text{-}cl\text{-}RA) \subset \mathcal{L}((k+1)\text{-}cl\text{-}RA)$, for all $k \geq 1$.
  - **Note**: The following language: $\{ (c^k a c^k)^n (c^k b c^k)^n \mid n \geq 0 \}$ belongs to $\mathcal{L}((k+1)\text{-}cl\text{-}RA) - \mathcal{L}(k\text{-}cl\text{-}RA)$.

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  - For each accepted $z \in \Sigma^{<n} - \{\lambda\}$ we add instruction $i_z = (¢, z, \$)$.

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  - **Note**: L_1 can be recognized by a simple *RRWW*-automaton. Moreover L_1 is a *context-free language*, thus we get the following corollary:
- Corollary:
  - $\mathcal{L}(cl\text{-}RA) \subset \mathcal{L}(RRWW)$.
  - CFL - $\mathcal{L}(cl\text{-}RA) \neq \emptyset$.

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- Let $L_2 = \{a^n b^n \mid n \geq 0\}$ and $L_3 = \{a^n b^{2n} \mid n \geq 0\}$ be two sample languages. Apparently both L_2 and L_3 are recognized by *1-cl-RA*-automata.

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- Theorem: Languages $L_2 \cup L_3$ and $L_2 . L_3$ are not recognized by any *cl-RA*-automaton.
- Corollary: $\mathcal{L}(cl\text{-}RA)$ is not closed under union, concatenation, and homomorphism.
  - For homomorphism use $\{a^n b^n \mid n \geq 0\} \cup \{c^n d^{2n} \mid n \geq 0\}$ and homomorphism defined as: $a \mapsto a$, $b \mapsto b$, $c \mapsto a$, $d \mapsto b$. ■

Some Theorems

- It is easy to see that each of the following languages:
  - $L_4 = \{a^n c b^n \mid n \geq 0\} \cup \{a^m b^m \mid m \geq 0\}$
  - $L_5 = \{a^n c b^m \mid n, m \geq 0\} \cup \{\lambda\}$
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- <u>Corollary</u>: $\mathcal{L}(cl\text{-}RA)$ is not closed under:
  - intersection: $L_1 = L_4 \cap L_5$.
  - intersection with regular language: L_5 is regular.
  - set difference: $L_1 = (L_4 - L_6) \cup \{\lambda\}$.

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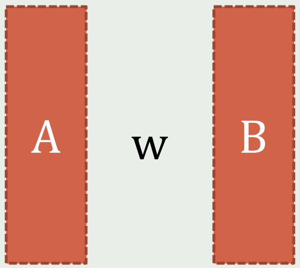
  - Note: We use the following notation for the (A, w, B):



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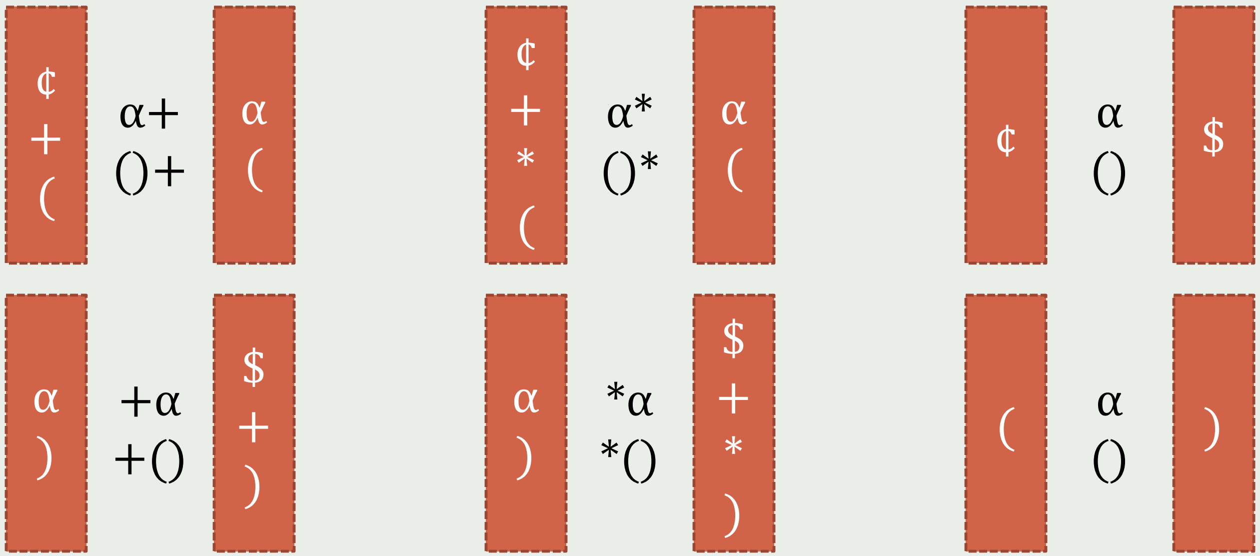
- The following *1-cl-RA*-automaton is sufficient:



Arithmetic Expressions - Example

Expression	Instruction
α*α + ((α + α) + (α + α*α))*α	(¢, α*, α)
α + ((α + α) + (α + α*α))*α	(α, +α,))
α + ((α) + (α + α*α))*α	( ), *α, $)
α + ((α) + (α + α*α))	(+, α*, α)
α + ((α) + (α + α))	((, α+, α)
α + ((α) + (α))	((, α,))
α + (( ) + (α))	((, ( )+, ( )
α + ((α))	((, α,))
α + (( ))	((, ( ),))
α + ( )	(¢, α+, ( )
( )	(¢, ( ), $)
λ	accept

Nondeterminism

- Assume the following instructions:
  - *R1 = (bb, a, bbbb)*
  - *R2 = (bb, bb, $)*
  - *R3 = (¢, cbb, $)*

  and the word: *cbbabbbb*.

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- Assume the following instructions:
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  and the word: *cbbabbbb*. Then:
  - $cbb\underline{a}bbbb \vdash^{R1} cbbbb\underline{bb} \vdash^{R2} cbb\underline{bb} \vdash^{R2} \underline{cbb} \vdash^{R3} \lambda$.

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  and the word: *cbbabbbb*. Then:
  - *cbbabbbb* $\vdash^{R1}$ *cbbbbbb* $\vdash^{R2}$ *cbbbb* $\vdash^{R2}$ *cbb* $\vdash^{R3}$ λ.
- **But** if we have started with *R2*:
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  then it would not be possible to continue.

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  and the word: *cbbabbbb*. Then:
  - $cbb\underline{a}bbbb \vdash^{R1} cbbbb\underline{bb} \vdash^{R2} cbb\underline{bb} \vdash^{R2} \underline{cbb} \vdash^{R3} \lambda$.
- **But** if we have started with *R2*:
  - $cbbabb\underline{bb} \vdash^{R2} cbbabb$

  then it would not be possible to continue.
- ⇒ The order of used instructions is important!

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- As we have seen not all context-free languages are recognized by a *cl-RA*-automaton.

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- As we have seen not all context-free languages are recognized by a *cl-RA*-automaton.
- We still can characterize CFL using clearing restarting automata, inverse homomorphism and Greibach's hardest context-free language.

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- Greibach constructed a context-free language *H*, such that:
  - Any context-free language can be parsed in whatever time or space it takes to recognize *H*.

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  - Any context-free language can be parsed in whatever time or space it takes to recognize *H*.
  - Any context-free language L can be obtained from *H* by an inverse homomorphism. That is, for each context-free language *L*, there exists a homomorphism φ: $L = \varphi^{-1}(H)$.

Greibach's Hardest CFL

- By *S. A. Greibach*, definition from *Section 10.5* of *M. Harrison, Introduction to Formal Language Theory, Addison-Wesley, Reading, MA, 1978.*
- Let $\Sigma = \{a_1, a_2, \underline{a}_1, \underline{a}_2, \#, c\}, d \notin \Sigma$.

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- Let $\Sigma = \{a_1, a_2, \underline{a}_1, \underline{a}_2, \#, c\}, d \notin \Sigma$.
- Let D_2 be *Semi-Dyck language* on $\{a_1, a_2, \underline{a}_1, \underline{a}_2\}$ generated by the grammar: $S \rightarrow \lambda \mid SS \mid a_1 S \underline{a}_1 \mid a_2 S \underline{a}_2$.

Greibach's Hardest CFL

- By *S. A. Greibach*, definition from *Section 10.5* of *M. Harrison, Introduction to Formal Language Theory, Addison-Wesley, Reading, MA, 1978.*
- Let $\Sigma = \{a_1, a_2, \underline{a}_1, \underline{a}_2, \#, c\}$, $d \notin \Sigma$.
- Let D_2 be *Semi-Dyck language* on $\{a_1, a_2, \underline{a}_1, \underline{a}_2\}$ generated by the grammar: $S \rightarrow \lambda \mid SS \mid a_1 S \underline{a}_1 \mid a_2 S \underline{a}_2$.
- Then $H = \{\lambda\} \cup \{\prod_{i=1..n} x_i c y_i c z_i d \mid n \geq 1, y_1 y_2 \ldots y_n \in \#D_2, x_i, z_i \in \Sigma^*\}$,
  - $y_1 \in \# . \{a_1, a_2, \underline{a}_1, \underline{a}_2\}^*$,
  - $y_i \in \{a_1, a_2, \underline{a}_1, \underline{a}_2\}^*$ for all $i > 1$.

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- Cherubini et. al defined *H* using *associative language description (ALD)* which uses one auxiliary symbol.

  (*in Associative language descriptions, Theoretical Computer Science, 270 (2002), 463-491*)
- So we will slightly extend the definition of *cl-RA*-automata in order to be able to recognize more languages including *H*.

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  - where $x \in LC_k$, $y \in RC_k$, $z \in \Gamma^+$.
    - left context $LC_k = \Gamma^k \cup ¢.\Gamma^{\leq k-1}$
    - right context $RC_k = \Gamma^k \cup \Gamma^{\leq k-1}.\$$

Definition

- A word $w = uzv$ can be *rewritten* to usv ($u\underline{z}v \vdash_M usv$) if and only if there exist an instruction $i = (x, z \rightarrow s, y) \in I$ such that:
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- The *k-Δcl-RA*-automaton M *recognizes* the language $L(M) = \{w \in \Sigma^* \mid M \text{ accepts } w\}$.

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- $\mathcal{L}(\Delta cl\text{-}RA) = \bigcup_{k \geq 1} \mathcal{L}(k\text{-}\Delta cl\text{-}RA)$.
- **Note**: For every $\Delta cl\text{-}RA$ M: $\lambda \vdash^{*}_{M} \lambda$ hence $\lambda \in L(M)$. If we say that $\Delta cl\text{-}RA$ M recognizes a language L, we mean that $L(M) = L \cup \{\lambda\}$.

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- For instance:
  - $aaacbbb \vdash^{Rc1} aa\Delta bb \vdash^{R\Delta 1} \underline{a\Delta b} \vdash^{R\Delta 2} \lambda$.
- Now we see that the word *aaacbbb* is accepted because $aaacbbb \vdash^{*}_{M} \lambda$.

Back to Greibach's Hardest CFL

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  - In the *second phase* we check if $y_1y_2 \ldots y_n \in \#D_2$.

Instructions recognizing Hardest CFL H

- Suppose $\Sigma = \{a_1, a_2, \underline{a}_1, \underline{a}_2, \#, c\}$, $d \notin \Sigma$, $\Gamma = \Sigma \cup \{d, \Delta\}$.

Instructions for the first phase:	Instructions for the second phase:
(1) $(¢, \Sigma \to \lambda, \Sigma)$	(7) $(\Gamma, a_1\underline{a}_1 \to \lambda, \Gamma - \{\#\})$
(2) $(\Sigma, \Sigma \to \lambda, d)$	(8) $(\Gamma, a_2\underline{a}_2 \to \lambda, \Gamma - \{\#\})$
(3) $(d, \Sigma \to \lambda, \Sigma)$	(9) $(\Gamma, a_1\Delta\underline{a}_1 \to \Delta, \Gamma - \{\#\})$
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- In fact, there is no such thing as a *first phase* or a *second phase*. We have only instructions.
- Theorem: $H \subseteq L(M)$, $H \supseteq L(M)$.

Learning Clearing Restarting Automata

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Step 1: $k := 1$.

Step 2: For each reduction $u_i \vdash_M v_i$ choose (nondeterministically) a factorization of u_i, such that $u_i = x_i z_i y_i$ and $v_i = x_i y_i$.

Learning Clearing Restarting Automata

Step 3: Construct a k-cl-RA-automaton $M = (\Sigma, I)$, where $I = \{ (Suff_k(¢.x_i), z_i, Pref_k(y_i.\$)) \mid i = 1, \ldots, n \}$.

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<u>Step 5</u>: If the automaton passed all the tests, return M. Otherwise try another factorization of the known reductions and continue by Step 3 or increase k and continue by Step 2.

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Learning Clearing Restarting Automata

- Even if the algorithm is very simple, it can be used to infer some non-trivial clearing (and after some generalization also Δ-clearing) restarting automata.
- Although Δ-clearing restarting automata are stronger than clearing restarting automata, we will see that even clearing restarting automata can recognize some *non-context-free languages*.
- However, it can be shown, that:
- Theorem: $\mathcal{L}(\Delta cl\text{-}RA) \subseteq$ CSL, where CSL denotes the class of *context-sensitive languages*.

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- <u>Theorem</u>: There exists a *k-cl-RA*-automaton *M* recognizing a language that is not context-free.
  - **Idea**. We try to create a *k-cl-RA*-automaton *M* such that $L(M) \cap \{(ab)^n \mid n>0\} = \{(ab)^{2^m} \mid m\geq 0\}$.
  - If *L(M)* is a CFL then the intersection with a regular language is also a CFL. In our case the intersection is not a CFL.

Learning Non-Context-Free Language

- Example:

  ¢ abababababababab $

Learning Non-Context-Free Language

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  $¢\ ababababababab\underline{a}b\ \$ \vdash_M ¢\ ababababab\underline{a}babb\ \$ \vdash_M$

  $¢\ ababab\underline{a}babbabb\ \$ \vdash_M ¢\ ab\underline{a}babbabbabb\ \$ \vdash_M$

  $¢\ abbabbabbabb\ \$$

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¢ $abab$ \$

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- From this sample computation we can collect 15 reductions with unambiguous factorizations and use them as an input to our algorithm.

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Learning Non-Context-Free Language

- The only variable we have to choose is *k* - the length of the context of the instructions.
- For $k = 1$ we get the following set of instructions:

  (b, a, b), (a, b, b), (¢, ab, $)

Learning Non-Context-Free Language

- The only variable we have to choose is k - the length of the context of the instructions.
- For $k = 1$ we get the following set of instructions:

  $(b, \underline{a}, b), (a, \underline{b}, b), (¢, \underline{ab}, \$)$

  But then the automaton would accept the word *ababab* which does not belong to L:

  $abab\underline{a}b \vdash_M ab\underline{a}bb \vdash_M a\underline{b}bb \vdash_M a\underline{b}b \vdash_M \underline{ab} \vdash_M \lambda.$

Learning Non-Context-Free Language

- For $k = 2$ we get the following set of instructions:

  $(ab, \underline{a}, \{b\$, ba\}), (\{¢a, ba\}, \underline{b}, \{b\$, ba\}), (¢, \underline{ab}, \$)$

Learning Non-Context-Free Language

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Learning Non-Context-Free Language

- For *k = 2* we get the following set of instructions:

  (ab, <u>a</u>, {b\$, ba}), ({¢a, ba}, <u>b</u>, {b\$, ba}), (¢, <u>ab</u>, \$)

  But then the automaton would accept the word *ababab* which does not belong to *L*:

  $aba\underline{b}ab \vdash_M aba\underline{b}b \vdash_M ab\underline{a}b \vdash_M a\underline{b}b \vdash_M \underline{ab} \vdash_M \lambda.$

- For *k = 3* we get the following set of instructions:

  ({¢ab, bab}, <u>a</u>, {b\$, bab}), ({¢a, bba}, <u>b</u>, {b\$, bab}), (¢, <u>ab</u>, \$)

Learning Non-Context-Free Language

- For $k = 2$ we get the following set of instructions:

  $(ab, \underline{a}, \{b\$, ba\}), (\{\text{¢}a, ba\}, \underline{b}, \{b\$, ba\}), (\text{¢}, \underline{ab}, \$)$

  But then the automaton would accept the word *ababab* which does not belong to L:

  $abab\underline{a}b \vdash_M aba\underline{b}b \vdash_M ab\underline{a}b \vdash_M a\underline{b}b \vdash_M \underline{ab} \vdash_M \lambda.$

- For $k = 3$ we get the following set of instructions:

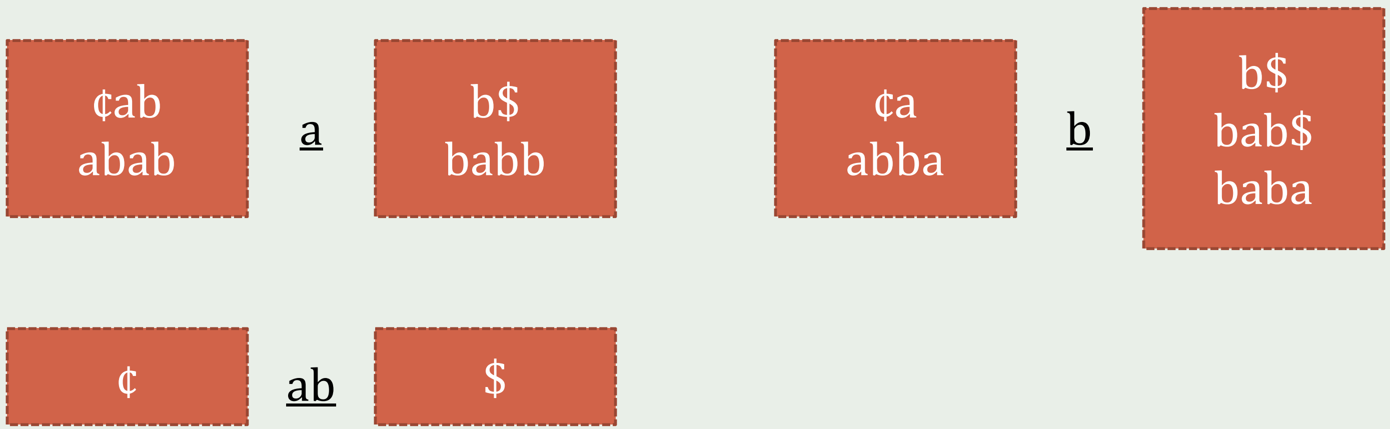
  $(\{\text{¢}ab, bab\}, \underline{a}, \{b\$, bab\}), (\{\text{¢}a, bba\}, \underline{b}, \{b\$, bab\}), (\text{¢}, \underline{ab}, \$)$

  And again we get:

  $ab\underline{a}bab \vdash_M aba\underline{b}b \vdash_M ab\underline{a}b \vdash_M a\underline{b}b \vdash_M \underline{ab} \vdash_M \lambda.$

Learning Non-Context-Free Language

- Finally, for *k = 4* we get the required *4-cl-RA*-automaton *M*.



Learning Non-Context-Free Language

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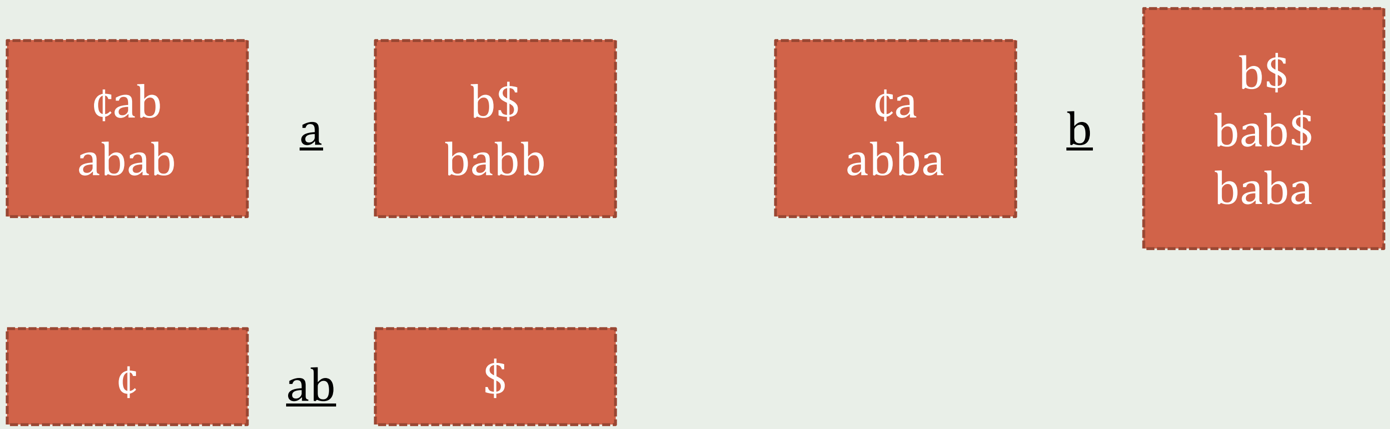

- For this *4-cl-RA*-automaton *M* it can be shown, that:
$L(M) \cap \{(ab)^n \mid n>0\} = \{(ab)^{2^m} \mid m\geq 0\}$.

Conclusion

- We have seen that knowing some sample computations (or even reductions) of a *cl-RA*-automaton (or *Δcl-RA*-automaton) it is extremely simple to infer its instructions.

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Conclusion

- We have seen that knowing some sample computations (or even reductions) of a *cl-RA*-automaton (or *Δcl-RA*-automaton) it is extremely simple to infer its instructions.
- The instructions of a *Δcl-RA*-automaton are human readable which is an advantage for their possible applications e.g. in linguistics.
- Unfortunately, we still do not know whether *Δcl-RA*-automata can recognize all context-free languages.

Conclusion

- If we generalize *Δcl-RA*-automata by enabling them to use any number of auxiliary symbols: $\Delta_1, \Delta_2, \ldots, \Delta_n$ instead of single Δ, we will increase their power up-to context sensitive languages.

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  - Such automata can easily accept all languages generated by context-sensitive grammars with productions in *one-sided normal form*: $A \rightarrow a, A \rightarrow BC, AB \rightarrow AC$

    where A, B, C are nonterminals and a is a terminal.

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  - Such automata can easily accept all languages generated by context-sensitive grammars with productions in *one-sided normal form*: $A \rightarrow a$, $A \rightarrow BC$, $AB \rightarrow AC$

    where A, B, C are nonterminals and a is a terminal.
  - Penttonen showed that for every context-sensitive grammar there exists an equivalent grammar in one-sided normal form.

Open Problems

- What is the difference between language classes of $\mathcal{L}(k\text{-}cl\text{-}RA)$ and $\mathcal{L}(k\text{-}\Delta cl\text{-}RA)$ for different values of k?

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Open Problems

- What is the difference between language classes of $\mathcal{L}(k\text{-}cl\text{-}RA)$ and $\mathcal{L}(k\text{-}\Delta cl\text{-}RA)$ for different values of k?
- Can $\Delta cl\text{-}RA$-automata recognize all string languages defined by ALD's?
- What is the relation between $\mathcal{L}(\Delta cl\text{-}RA)$ and the class of one counter languages, simple context-sensitive grammars (they have single nonterminal), etc?

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